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389. Proposed by FRANK R. MORRIS, Glendale, Calif.

A man is at the southeast corner of a section of land and wishes to walk to the opposite corner in the least possible time. A circular track with a radius of $1/\pi$ miles is located in the section tangent to the west line at a point 120 rods from the south line. Conditions are such that he can walk at the rate of 4 miles an hour inside the track and 3 miles an hour outside the track. What course should he choose and how long is it?

MECHANICS.

When this issue was made up, no solutions had been received for numbers 274, 277, 279, 287, 291, and 292.

311. Proposed by B. J. BROWN, Student in Drury College.

A particle oscillates in a straight line about a center of force varying as the distance, and is subject to a retardation $k \times (\text{vel.})^2$. If a, b be two successive elongations, on opposite sides, prove that $(1+2ka)e^{-2ka} = (1-2kb)e^{2kb}$. What form does the result take if a is infinite? From *Lamb's Dynamics*, p. 299, ex. 14.

312. Proposed by C. N. SCHMALL, New York City.

A ball of elasticity e is projected upward from a point on an inclined plane, so that after its first contact with the plane it rebounds to its starting point. If ϕ be the inclination of the plane to the horizontal, and ψ the angle made by the line of projection with the inclined plane, show that

$$\cot\phi \cot\psi = e + 1.$$

313. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A heavy extensible wire of length c and of constant cross-section w, and density k, is suspended by one end and hangs vertically. If e is the coefficient of elasticity, show that the length of the wire when stretched will be $c(1 + ekgw/2)$.

NUMBER THEORY.

When this issue was made up, no solutions had been received for numbers 188–9, 191–2, 196, 200, 205, 208–9, 211, 214–15, 217, and 219.

232. Proposed by E. T. BELL, Seattle, Washington.

If $F(x)$ is any function of x which vanishes with x, and which, for $0 < |x| \leqq |\xi|$, can be expanded in an absolutely convergent series of positive powers of x, show that a function $f(n)$ may be found, essentially in one way only, such that

$$\int_0^{\xi} \frac{1}{x} F(x)dx = -\log \prod_{n=1}^{\infty} (1-\xi^n)^{(1/n)f(n)},$$

and find the form of $f(n)$ explicitly in terms of the coefficients in the expansion of $F(x)$. Hence, as particular examples, expand (when possible by this method) e^x as an infinite product, and show that

$$\frac{1}{e} = \prod_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)^{(1/n)\phi(n)}$$

where $\varphi(n)$ is the totient of n.

233. Proposed by V. M. SPUNAR, Chicago, Illinois.

Solve in rational numbers $x^2 + y^2 = a^2$, $xy = m/n$, where m and n are integers and relatively prime to each other.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

424. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{n}{a} - \binom{n-1}{a}\binom{i}{1} + \binom{n-2}{a}\binom{i}{2} - \binom{n-3}{a}\binom{i}{3} + \cdots + (-1)^i\binom{n-i}{a}$$

and consider the cases $i = a$ and $i > a$.

I. Solution by Frank Irwin, University of California.

The sum is $\binom{n-i}{a-i}$. This will hold for all (positive) values of n, a, i, provided we put $\binom{k}{l} = 0$ if $k < l$ or if l is negative. Thus for $i > a$, the sum will be zero; while for $n - i < a$, some of the terms of the sum will drop out.

First Proof.—These statements may be proved by mathematical induction. For they are true when $i = 1$; suppose them then to be true for some particular value of i and all values of n. We shall show that they are true for $i + 1$. For we have:

$$\binom{n}{a} - \binom{n-1}{a}\binom{i}{1} + \cdots \pm \binom{n-k}{a}\binom{i}{k} + \cdots = \binom{n-i}{a-i},$$

$$\binom{n-1}{a} - \binom{n-2}{a}\binom{i}{1} + \cdots \mp \binom{n-k}{a}\binom{i}{k-1} + \cdots = \binom{n-i-1}{a-i};$$

whence, subtracting, we obtain

$$\binom{n}{a} - \binom{n-1}{a}\binom{i+1}{1} + \cdots \pm \binom{n-k}{a}\left[\binom{i}{k} + \binom{i}{k-1}\right] + \cdots$$
$$= \binom{n-i}{a-i} - \binom{n-i-1}{a-i},$$

or

$$\binom{n}{a} - \binom{n-1}{a}\binom{i+1}{1} + \cdots \pm \binom{n-k}{a}\binom{i+1}{k} + \cdots = \binom{n-i-1}{a-i-1},$$

the desired result.

Second Proof.—$\binom{n-i}{a-i}$ gives the combinations of $n - i$ objects, say white balls, $a - i$ at a time. We may count these combinations in another way. Let us add to the white balls i black balls, and consider the $\binom{n}{a}$ combinations of black and white balls together, a at a time. If we take one of these combinations *that contains all the black balls* and discard the latter, we get one of the combinations of the white balls $a - i$ at a time; and each of these latter combinations may be obtained in this way. To count these latter, then, we must subtract from the $\binom{n}{a}$ combinations above those that omit any of the black balls. There are $\binom{n-1}{a}$ that omit a particular black ball, $\binom{i}{1}$ black balls; but the number

$$\binom{n}{a} - \binom{n-1}{a}\binom{i}{1}$$

is too small, since a combination that omits more than one black ball has been subtracted, in the second term, more than once. It is easily seen how, proceeding along these lines, we obtain our given sum

$$\binom{n}{a}-\binom{n-1}{a}\binom{i}{1}+\binom{n-2}{a}\binom{i}{2}-\cdots,$$

and it remains to examine how often herein a particular combination, say one that omits k black balls, has been reckoned. A little consideration will show that it has been counted $1-\binom{k}{1}+\binom{k}{2}-\cdots\pm 1=0$ times; so that the only combinations counted are those that contain all i black balls, and our sum is equal to the number of these combinations, that is, to $\binom{n-i}{a-i}$, as was to be proved.

When $i>a$, $a-i$ is negative and therefore $\binom{n-i}{a-i}=0$.

II. Solution by the Proposer.

By an elementary theorem in the calculus of finite differences, we have

$$\Delta^i u_x = u_{x+1}-\binom{i}{1}u_{x+i-1}+\binom{i}{2}u_{x+i-2}-\cdots+(-1)^i u_x.$$

Making here $x+i=n$, or $x=n-i$, we obtain

$$u_n-\binom{i}{1}u_{n-1}+\binom{i}{2}u_{n-2}-\cdots+(-1)^i u_{n-i}=\Delta^i u_{n-i}.$$

If we take

$$u_n=\binom{n}{a}, \text{ so that } u_{n-1}=\binom{n-1}{a},\ u_{n-2}=\binom{n-2}{a}, \text{ etc.,}$$

the last equation becomes

$$\binom{n}{a}-\binom{i}{1}\binom{n-1}{a}+\binom{i}{2}\binom{n-2}{a}-\cdots+(-1)^i\binom{n-i}{a}=\Delta^i\binom{n-i}{a},$$

the first member of which is the given series.

In order to evaluate the second member, we notice that

$$\Delta\binom{x}{a}=\binom{x+1}{a}-\binom{x}{a}.$$

But as

$$\binom{x+1}{a}-\binom{x}{a}=\frac{(x+1)!}{(x+1-a)!a!}-\frac{x!}{(x-a)!a!}=\frac{x!}{(x+1-a)!(a-1)!}=\binom{x}{a-1},$$

therefore

$$\Delta\binom{x}{a}=\binom{x}{a-1},\quad \Delta^2\binom{x}{a}=\Delta\binom{x}{a-1}=\binom{x}{a-2},\quad \Delta^3\binom{x}{a}=\binom{x}{a-3},\quad \text{etc.}$$

and in general

$$\Delta^i\binom{x}{a}=\binom{x}{a-i};\quad \Delta^i\binom{n-i}{a}=\binom{n-i}{a-i}.$$

Hence

$$\binom{n}{a}-\binom{i}{1}\binom{n-1}{a}+\binom{i}{2}\binom{n-2}{a}-\cdots+(-1)^i\binom{n-i}{a}=\binom{n-i}{a-i}. \qquad (1)$$

In the case when $i=a$, we have $a-i=0$, and $\binom{n-i}{a-i}=1$; therefore, equation (1) becomes

$$\binom{n}{a}-\binom{a}{1}\binom{n-1}{a}+\binom{a}{2}\binom{n-2}{a}-\cdots+(-1)^a\binom{n-a}{a}=1. \qquad (2)$$

When $i>a$, then $a-i$ is negative, so that $\binom{n-i}{a-i}=0$. Hence for $i>a$,

$$\binom{n}{a}-\binom{i}{1}\binom{n-1}{a}+\binom{i}{2}\binom{n-2}{a}-\cdots+(-1)^i\binom{n-i}{a}=0. \qquad (3)$$

An excellent solution by a somewhat different method from the ones exhibited in these two solutions was received from A. M. HARDING.

425. Proposed by CLIFFORD N. MILLS, Brookings, S. D.

Solve for x and y the equations $2^{x+y}=6$, $2^{x+1}=3^y$.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We shall agree to mean by a^x, where a is real, $e^{x\log a}$, in which the real logarithm of a is to be taken, so that a^x is single valued. We have, then, from the given equations, $(x+y)\log 2=\log 6$, $(x+1)\log 2=y\log 3$, where all the logarithms are real. Solving these linear equations for x and y, we obtain the required solutions

$$x=\frac{(\log 6)^2-\log 2\cdot\log 12}{\log 2\log 6}=1.198 \text{ and } y=\frac{\log 12}{\log 2}=1.387.$$

On account of the homogeneity of these fractions in the logarithmic function, we may substitute logarithms to the base 10 in our computation of the numerical values of x and y.

Also solved by EMMA GIBSON, NATHAN ALTSHILLER, A. L. MCCARTY, A. M. HARDING, HORACE OLSON, RICHARD MORRIS, G. W. HARTWELL, C. E. GITHENS, FRANK IRVIN, ELIZABETH B. DAVIS, F. L. CARMICHAEL, EDWARD S. INGHAM, S. A. JOFFE, W. C. EELLS, ELMER SCHUYLER, V. M. SPUNAR, R. M. MATHEWS, and CYRIL A. NELSON.

GEOMETRY.

454. Proposed by LOUIS ROUILLION, Mechanics Institute, New York City.

Show how to construct an equilateral triangle with its vertices lying on three [parallel] lines not equally spaced.

I. SOLUTION BY C. N. SCHMALL, New York City.

The proposer evidently meant that the three lines are parallel.

(i) Let x, y, and z be the three parallel lines in order. At any point, M, on the line y, construct the angles AMC and BMC each equal to 60° (Fig. 1). Draw the line AB, and, about the triangle AMB, circumscribe the circle ABC.